Soluzioni della serie5_03

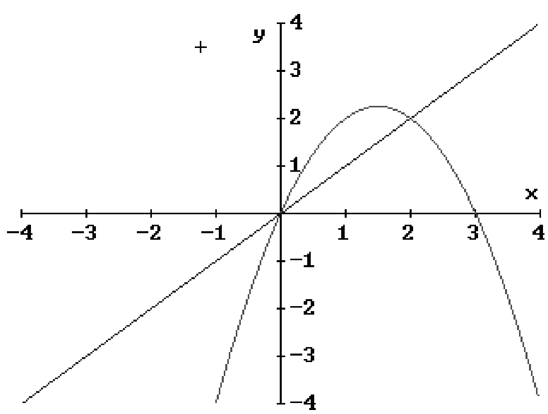
1. $D_f=\mathbb{R}-\left\{\frac{3}{2}\right\}$ posto $\frac{x}{2}=t \quad x=2t \quad dx=2dt$ per $x\to 0 \quad t\to 0$ e per $x\to 1 \quad t\to \frac{1}{2}$ quindi $\int_0^1 f\left(\frac{x}{2}\right)=\int_0^{\frac{1}{2}}\frac{(2t)^2-4(2t)+4}{3-4t}\cdot 2dt=$

   $=8\int_0^{\frac{1}{2}}\frac{t^2-2t+1}{3-4t}\,dt=8\int_0^{\frac{1}{2}}\left(-\frac{1}{4}t+\frac{5}{16}-\frac{1}{16}\cdot\frac{1}{4t-3}\right)dt=8\left[-\frac{1}{8}t^2+\frac{5}{16}t-\frac{1}{64}\ln|4t-3|\right]_0^{\frac{1}{2}}=$

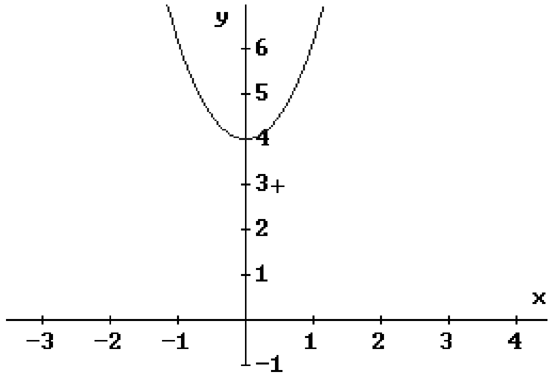
   $=1-\frac{1}{8}\ln 3$

2. Essendo $y=\frac{ax+b}{x^2};\quad y'=-\frac{ax+2b}{x^3};\quad y''=\frac{2ax+6b}{x^6}$ consideriamo il sistema: $\begin{cases}y(2)=\frac{1}{4}\\ y''(2)=0\end{cases} \Rightarrow \begin{cases}\frac{2a+b}{4}=\frac{1}{4}\\ \frac{4a+6b}{64}=0\end{cases}$ e ricaviamo $\begin{cases}a=1\\ b=-1\end{cases}\Rightarrow \quad y=\frac{x-1}{x^2}$

3. determiniamo la regione di piano mediante $\begin{cases}y=-x^2+3x\\ y=x\end{cases} \Rightarrow \begin{cases}x=0; \quad x=2\\ y=0; \quad y=2\end{cases}$ e calcoliamo: $V=\pi\int_0^2\left(-x^2+3x\right)^2dx-\pi\int_0^2 x^2dx=\int_0^2\left(x^4-6x^3+9x^2-x^2\right)dx=\frac{56}{15}\pi$



4. poiché $y=a\,e^x+b\,e^{-x}$ e $y'=a\,e^x-b\,e^{-x}$ consideriamo il sistema:
   $\begin{cases}y(0)=4\\ y'(0)=0\end{cases} \Leftrightarrow \begin{cases}a+b=4\\ a-b=0\end{cases} \begin{cases}a=2\\ b=2\end{cases}$ quindi $y=2\,e^x+2\,e^{-x}$



e possiamo facilmente verificare che $f(-x) = f(x)$

5. $p(E_1) = p(E_2) = p(E_3) = \frac{3}{6} = \frac{1}{2} \quad \Rightarrow \quad p = \frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2} = \frac{1}{8}$

6. $\lim_{x \to 1^-} (3x+2) = \lim_{x \to 1^+} (a\,x^2 - 4) \quad \Rightarrow \quad 5 = a - 4 \quad \Rightarrow \quad a = 9 \qquad f(x) = \begin{cases} 3x+2 & x < 1 \\ 9x^2 - 4 & x \geq 1 \end{cases}$

quindi: $\int_0^2 f(x)\,dx = \int_0^1 (3x+2)\,dx + \int_1^2 (9x^2 - 4)\,dx = \left[3 \cdot \frac{x^2}{2} + 2x\right]_0^1 + \left[9 \cdot \frac{x^3}{3} - 4x\right]_1^2 = \frac{41}{2}$